AC 26/2/2015

Item No. 4.30

UNIVERSITY OF MUMBAI

Syllabus for: S. Y. B. Sc. /S. Y. B. A.

Program: B.Sc. /B.A.

Course: Mathematics

(Credit Based Semester and Grading System with effect from the Academic year (2015–2016)

SEMESTER III

CALCULUS III				
Course Code	UNIT	TOPICS	Credits	L/ Week
USMT 301	I	Riemann Integration	3	3
	II	Indefinite and improper integrals		
	III	Applications		
ALGEBRA III				
USMT 302 UAMT 301	I	Linear Transformations and Matrices	3	3
	II	Determinants		
	III	Inner Product Spaces		
DISCRETE MATHEMATICS				
USMT 303 UAMT 302	I	Preliminary Counting	3	3
	II	Advanced Counting		
	III	Permutations and Recurrence relation		

SEMESTER IV

CALCULUS OF SEVERAL VARIABLES				
Course Code	UNIT	TOPICS	Credits	L/ Week
USMT 401	I	Functions of several variables	3	3
	II	Differentiation		
	III	Applications		
ALGEBRA IV				
USMT 402 UAMT 401	I	Groups and Subgroups	3	3
	II	Cyclic Groups and Cyclic Subgroups		
	III	Lagrange's Theorem and Group homomorphism		
ORDINARY DIFFERENTIAL EQUATIONS				
USMT 403 UAMT 402	I	First order First degree Differential equations	3	3
	II	Second order Linear Differential equations		
	III	Numerical Solution for Ordinary Differential Equations		

Teaching Pattern

1. Three lectures per week per course.
2. One Tutorial per week per batch per course (The batches to be formed as prescribed by the University).
3. One assignment per course or one project.

S. Y. B. Sc./ S. Y. B. A. Mathematics

SEMESTER III
USMT 301: CALCULUS III

Note: All topics have to be covered with proof in details (unless mentioned otherwise) and examples.

Unit I: Riemann Integration (15 Lectures)

Approximation of area, Upper / Lower Riemann sums and properties, Upper / Lower integrals, Definition of Riemann integral on a closed and bounded interval, Criterion for Riemann integrability, if $a < c < b$ then $f \in R[a,b]$ if and only if $f \in R[a,c]$ and $f \in R[c,b]$ and $\int_a^b f = \int_a^c f + \int_c^b f$. Properties: if $f,g \in R[a,b] \Rightarrow f+g,\ \lambda f \in R[a,b]$ and $\int_a^b (f+g) = \int_a^b f + \int_a^b g$, $\int_a^b \lambda f = \lambda \int_a^b f$, $f \in R[a,b] \Rightarrow |f| \in R[a,b]$ and $|\int_a^b f| \le \int_a^b |f|$, $f \ge 0 \Rightarrow \int_a^b f \ge 0$,, $f \in C[a,b] \Rightarrow f \in R[a,b]$, if f is bounded with finite number of discontinuities then $f \in R[a,b]$, generalize this if f is monotone then $f \in R[a,b]$.

Unit II: Indefinite and improper integrals (15 Lectures)

Continuity of $F(x) = \int_a^x f(t)dt$ where $f \in R[a,b]$, Fundamental theorem of calculus, Mean value theorem, Integration by parts, Leibnitz rule, Improper integrals- type 1 and type 2, Absolute convergence of improper integrals, Comparison tests, Abel's and Dirichlet's tests (without proof) β and Γ functions and their properties, relationship between β and Γ functions.

Unit III: Applications (15 Lectures)

Topics from analytic geometry- sketching of regions in R^2 and R^3, graph of a function, level sets, cartesian coordinates, polar coordinates, spherical coordinates, cylindrical coordinates and conversions from one coordinate system to another.

(a) Double integrals: Definition of double integrals over rectangles, properties, double integrals over a bounded region.

(b) Fubini theorem (without proof) - iterated integrals, double integrals as volume.

(c) Application of double integrals: average value, area, moment, center of mass.

(d) Double integral in polar form.

(Reference for Unit III: Sections 5.1, 5.2, 5.3 and 5.5 from Marsden-Tromba-Weinstein).

References:

(1) R. R. Goldberg, Methods of Real Analysis, Oxford and IBH, 1964.

(2) Ajit Kumar, S. Kumaresan, A Basic Course in Real Analysis, CRC Press, 2014.

(3) T. Apostol, Calculus Vol.2, , John Wiley.

(4) J. Stewart, Calculus, Brooke/Cole Publishing Co, 1994.

(5) J. E. Marsden, A. J. Tromba and A. Weinstein, Basic multivariable calculus.

(6) Bartle and Sherbet, Real analysis.

Suggested Tutorials:

1. Calculation of upper sum, lower sum and Riemann integral.
2. Problems on properties of Riemann integral.
3. Problems on fundamental theorem of calculus, mean value theorems, integration by parts, Leibnitz rule.
4. Convergence of improper integrals, applications of comparison tests, Abel's and Dirichlet's tests, and functions.
5. Sketching of regions in R^2 and R^3, graph of a function, level sets, conversions from one coordinate system to another.
6. Double integrals, iterated integrals, applications to compute average value, area, moment, center of mass.

USMT 302/ UAMT 301: ALGEBRA III

Note: Revision of relevant concepts is necessary.

Unit 1: Linear Transformations and Matrices (15 Lectures)

1. Review of linear transformations: Kernel and image of a linear transformation, Rank-Nullity theorem (with proof), Linear isomorphisms, inverse of a linear isomorphism, Any n-dimensional real vector space is isomorphic to R^n.
2. The matrix units, row operations, elementary matrices, elementary matrices are invertible and an invertible matrix is a product of elementary matrices.
3. Row space, column space of an $m \times n$ matrix, row rank and column rank of a matrix, Equivalence of the row and the column rank, Invariance of rank upon elementary row or column operations.

4. Equivalence of rank of an $m \times n$ matrix A and rank of the linear transformation $L_A : R^n \to R^m$ $(L_A \ (X) = AX)$, The dimension of solution space of the system of linear equations $AX = 0$ equals $n - rank(A)$.
5. The solutions of non-homogeneous systems of linear equations represented by $AX = B$, Existence of a solution when $rank(A) = rank(A, B)$, The general solution of the system is the sum of a particular solution of the system and the solution of the associated homogeneous system.

Reference for Unit 1: Chapter VIII, Sections 1, 2 of Introduction to Linear Algebra, Serge Lang, Springer Verlag and Chapter 4, of Linear Algebra A Geometric Approach, S. Kumaresan, Prentice-Hall of India Private Limited, New Delhi.

Unit II: Determinants (15 Lectures)

1. Definition of determinant as an n-linear skew-symmetric function from $R^n \times R^n \times R^n \times \ \ldots \ \times R^n \to R$ such that determinant of $(E^1, E^2, \ \ldots \ , E^n)$ is 1, where E^j denotes the j^{th} column of the $n \times n$ identity matrix I_n. Determinant of a matrix as determinant of its column vectors (or row vectors).
2. Existence and uniqueness of determinant function via permutations, Computation of determinant of $2 \times 2, 3 \times 3$ matrices, diagonal matrices, Basic results on determinants such as $\det(A^t) = \det(A)$, $\det(AB) = \det(A)\det(B)$, Laplace expansion of a determinant, Vandermonde determinant, determinant of upper triangular and lower triangular matrices.
3. Linear dependence and independence of vectors in R^n using determinants, The existence and uniqueness of the system $AX = B$, where A is an $n \times n$ matrix with $\det(A) \neq 0$, Cofactors and minors, Adjoint of an $n \times n$ matrix A, Basic results such as $A \, adj \, (A) = \det(A) \, I_n$. An $n \times n$ real matrix A is invertible if and only if $\det(A) \neq 0$, $A^{-1} = \frac{1}{\det(A)} \, adj \, (A)$ for an invertible matrix A, Cramer's rule.
4. Determinant as area and volume.

Reference for Unit 2: Chapter VI of Linear Algebra A geometric approach, S. Kumaresan, Prentice Hall of India Private Limited, 2001 and Chapter VII Introduction to Linear Algebra, Serge Lang, Springer Verlag.

Unit III: Inner Product Spaces (15 Lectures)

1. Dot product in R^n, Definition of general inner product on a vector space over R. Examples of inner product including the inner product $< f, g > = \int_{-\pi}^{\pi} f(t)g(t)dt$ on $C[-\pi,\pi]$, the space of continuous real valued functions on $[-\pi,\pi]$.
2. Norm of a vector in an inner product space. Cauchy-Schwarz inequality, Triangle inequality, Orthogonality of vectors, Pythagorus theorem and geometric applications in R^2, Projections on a line, The projection being the closest approximation, Orthogonal complements of a subspace, Orthogonal complements in R^2 and R^3. Orthogonal sets and orthonormal sets in an inner product space, Orthogonal and orthonormal bases. Gram-Schmidt orthogonalization process, Simple examples in R^3 , R^4.

Reference for Unit 3: Chapter VI, Sections 1, 2 of Introduction to Linear Algebra, Serge Lang, Springer Verlag and Chapter 5, of Linear Algebra A Geometric Approach, S. Kumaresan, Prentice-Hall of India Private Limited, New Delhi.

Recommended Books:

1. Serge Lang: Introduction to Linear Algebra, Springer Verlag.

2. S. Kumaresan: Linear Algebra A geometric approach, Prentice Hall of India Private Limited.

Additional Reference Books:

1. M. Artin: Algebra, Prentice Hall of India Private Limited.

2. K. Hoffman and R. Kunze: Linear Algebra, Tata McGraw-Hill, New Delhi.

3. Gilbert Strang: Linear Algebra and its applications, International Student Edition.

4. L. Smith: Linear Algebra, Springer Verlag.

5. A. Ramachandra Rao and P. Bhima Sankaran: Linear Algebra, Tata McGraw-Hill, New Delhi.

6. T. Banchoff and J. Wermer: Linear Algebra through Geometry, Springer Verlag Newyork, 1984.

7. Sheldon Axler : Linear Algebra done right, Springer Verlag, Newyork.

8. Klaus Janich : Linear Algebra.

9. Otto Bretcher: Linear Algebra with Applications, Pearson Education.

10. Gareth Williams: Linear Algebra with Applications, Narosa Publication.

Suggested Tutorials:

1. Rank-Nullity Theorem.

2. System of linear equations.
3. Determinants, calculating determinants of 2×2 matrices, $n \times n$ diagonal, upper triangular matrices using definition and Laplace expansion.
4. Finding inverses of $n \times n$ matrices using adjoint.
5. Inner product spaces, examples. Orthogonal complements in R^2 and R^3 .
6. Gram-Schmidt method.

USMT303/ UAMT302: DISCRETE MATHEMATICS

Unit I: Preliminary Counting (15 Lectures)

1. Finite and infinite sets, Countable and uncountable sets, examples such as $N, Z, N \times N, Q, (0,1), R$.
2. Addition and multiplication principle, Counting sets of pairs, two ways counting.
3. Stirling numbers of second kind, Simple recursion formulae satisfied by $S(n, k)$ and direct formulae for $S(n, k)$, for $k = 1,2, \ldots, n-1, n$.
4. Pigeon hole principle and its strong form, its applications to geometry, monotonic sequences.

Unit II: Advanced Counting (15 Lectures)

1. Binomial and Multinomial Theorem, Pascal identity, examples of standard identities such as the following with emphasis on combinatorial proofs.
   - $\sum_{k=0}^{r} \binom{m}{k}\binom{n}{r-k} = \binom{m+n}{r}$
   - $\sum_{i=r}^{n} \binom{i}{r} = \binom{n+1}{r+1}$
   - $\sum_{i=0}^{k} \binom{k}{i}^2 = \binom{2k}{k}$
   - $\sum_{i=0}^{n} \binom{n}{i} = 2^n$
2. Permutation and combination of sets and multi-sets, circular permutations, emphasis on solving problems.
3. Non-negative and positive integral solutions of equation $x_1 + x_2 + \cdots \cdots + x_k = n$.
4. Principle of Inclusion and Exclusion, its applications ,derangements, explicit formula for d_n, various identities involving d_n, deriving formula for Euler's phi function $\varphi(n)$.

Unit III: Permutations and Recurrence relation (15 Lectures)

1. Permutation of objects, $s_{n,}$ composition of permutations, results such as every permutation is product of disjoint cycles, every cycle is product of transpositions, even and odd permutations, rank and signature of permutation, cardinality of S_n, A_n.
2. Recurrence relation, definition of homogeneous, non-homogeneous, linear and non linear recurrence relation, obtaining recurrence relation in counting problems, solving (homogeneous as well as non homogenous) recurrence relation by using iterative method, solving a homogeneous recurrence relation of second degree using algebraic method proving the necessary result.

Recommended books:

1. Norman Biggs: Discrete Mathematics, Oxford University Press.
2. Richard Brualdi: Introductory Combinatorics, John Wiley and sons.
3. V. Krishnamurthy: Combinatorics – Theory and Applications, Affiliated East West Press.
4. Discrete Mathematics and its Applications, Tata McGraw Hills.
5. Schaum's outline series : Discrete mathematics,
6. Applied Combinatorics: Allen Tucker, John Wiley and Sons.

Suggested Practicals (Tutorials for B. A.):

1. Problems based on counting principles, Two way counting.
2. Stirling numbers of second kind, Pigeon hole principle.
3. Multinomial theorem, identities, permutation and combination of multi-set.
4. Inclusion-Exclusion principle, Euler phi function.
5. Derangement and rank signature of permutation.
6. Recurrence relation.

SEMESTER IV
USMT 401: CALCULUS OF SEVERAL VARIABLES

Note: All topics have to be covered with proof in details (unless mentioned otherwise) and examples.

Unit I: Functions of several variables (15 Lectures)

1. Euclidean space, R^n - norm, inner product, distance between two points, open ball in R^n, definition of an open set / neighborhood, sequences in R^n, convergence of sequences- these concepts should be specifically discussed for $n = 2$ and $n = 3$.
2. Functions from $R^n \rightarrow R$ (scalar fields) and from $R^n \rightarrow R^n$ (Vector fields). Iterated limits, limits and continuity of functions, basic results on limits and continuity of sum, difference, scalar multiples of vector fields, continuity and components of vector fields.
3. Directional derivatives and partial derivatives of scalar fields.
4. Mean value theorem for derivatives of scalar fields.

Reference for Unit I:

(1). T. Apostol, Calculus, Vol. 2, John Wiley.

(2) . J. Stewart, Calculus, Brooke/Cole Publishing Co.

Unit II: Differentiation (15 Lectures)

1. Differentiability of a scalar field at a point (in terms of linear transformation) and in an open set. Total derivative. Uniqueness of total derivative of a differentiable function at a point. (Simple examples of finding total derivative of functions such as $f(x, y) = x^2 + y^2$, $f(z, y, z) = x + y + z$, may be taken). Differentiability at a point implies continuity, and existence of direction derivative at the point. The existence of continuous partial derivatives in a neighborhood of point implies differentiability at the point.
2. Gradient of a scalar field, geometric properties of gradient, level sets and tangent planes.
3. Chain rule for scalar fields.
4. Higher order partial derivatives, mixed partial derivatives. Sufficient condition for equality of mixed partial derivative.

Reference for Unit II:

(1) Calculus, Vol. 2, T. Apostol, John Wiley.

(2) Calculus. J. Stewart. Brooke/Cole Publishing Co.

Unit III: Applications (15 Lectures)

1. Second order Taylor's formula for scalar fields.
2. Differentiability of vector fields, definition of differentiability of a vector field at a point Hessian /Jacobian matrix, differentiability of a vector field at a point implies continuity, the chain rule for derivative of vector fields statements only).
3. Mean value inequality.
4. Maxima, minima and saddle points.
5. Second derivative test for extrema of functions of two variables.
6. Method of Lagrange multipliers.

Reference for Unit III: sections 9.9, 9.10, 9.11, 9.12, 9.13, 9.14 from Apostol, Calculus Vol. 2.

(3) T. Apostol, Calculus, Vol. 2, John Wiley.

(4) J. Stewart, Calculus, Brooke/Cole Publishing Co.

Suggested Tutorials:

1. Sequences in $R^2 \ and \ R^3$, limits and continuity of scalar fields and vector fields, using "definition and otherwise, iterated limits.
2. Computing directional derivatives, partial derivatives and mean value theorem of scalar fields.
3. Total derivative, gradient, level sets and tangent planes.
4. Chain rule, higher order derivatives and mixed partial derivatives of scalar fields.
5. Taylor's formula, differentiation of a vector field at a point, finding Hessian/ Jacobean matrix, Mean value inequality.
6. Finding maxima, minima and saddle points, second derivative test for extrema of functions of two variables and method of Lagrange multipliers.

USMT 402 /UAMT 401: ALGEBRA IV

Unit I: Groups and Subgroups (15 Lectures)

(a) Definition of a group, abelian group, order of a group, finite and infinite groups.
Examples of groups including

- (i) $\mathbb{Z}, \mathbb{Q}, \mathbb{R}, \mathbb{C}$ under addition.
- (ii) $\mathbb{Q}^*(= \mathbb{Q}\setminus\{0\})$, $\mathbb{R}^*(= R\setminus\{0\})$, $\mathbb{C}^*(= C\setminus\{0\}$, $\mathbb{Q}^+(=$ positive rational numbers) under multiplication.
- (iii) $\mathbb{Z}_n$, the set of residue classes modulo n under addition.
- (iv) $U(n)$, the group of prime residue classes modulo n under multiplication.
- (v) The symmetric group S_n.
- (vi) The group of symmetries of a plane figure. The Dihedral group D_n as the group of symmetries of a regular polygon of n sides (for $n = 3, 4$).
- (vii) Klein 4-group.
- (viii) Matrix groups $M_{m\times n}(\mathbb{R})$ under addition of matrices $GL_n(\mathbb{R})$, the set of invertible real matrices, under multiplication of matrices.
- (ix) Examples such as S^1 as subgroup of C, μ_n the subgroup of n-th roots of unity.

Properties such as

1) In a group (G,.) the following indices rules are true for all integers n, m
   - i) $a^n a^m = a^{n+m}$ for all a in G.
   - ii) $(a^n)^m = a^{nm}$ for all a in G.
   - iii) $(ab)^n = a^n b^n$ for all a, b in G whenever $ab = ba$.
2) In a group $(G, .)$ the following are true
   - i) The identity element e of G is unique.
   - ii) The inverse of every element in G is unique.
   - iii) $(a^{-1})^{-1} = a$.
   - iv) $((a.b)^{-1} = b^{-1}.a^{-1}$.
   - v) If $a^2 = e$ for every a in G then $(G, .)$ is an abelian group.
   - vi) $(aba^{-1})^n = ab^n a^{-1}$ for every a, b in G and for every integer n.
   - vii) If $(a.b)^2 = a^2.b^2$ for every a, b in G then $(G, .)$ is an abelian group.
   - viii)$(\mathbb{Z}_n^*$, .) is a group if and only if n is a prime.
3) Properties of order of an element such as (n and m are integers)
   - i) If $o(a) = n$ then $a^m = e$ if and only if $n|m$.
   - ii) If $o(a) = nm$ then $o(a^n) = m$.
   - iii) If $o(a) = n$ then $o(a^m) = \frac{n}{(n,m)}$, where (n, m) is the GCD of n and m.
   - iv) $o(aba^{-1}) = o(b)$ and $o(ab) = o(ba)$.
   - v) If $o(a) = n$ and $o(b) = m$, $ab = ba$, $(n, m) = 1$ then $o(ab) = nm$.

(b) Subgroups

1) Definition, necessary and sufficient condition for a non-empty set to be a Subgroup.
2) The center $Z(G)$ of a group is a subgroup.
3) Intersection of two (or a family of) subgroups is a subgroup.
4) Union of two subgroups is not a subgroup in general. Union of two sub groups is a subgroup if and only if one is contained in the other.
5) If H and K are subgroups of a group G then HK is a subgroup of G if and

only if $HK = KH$.

Reference for Unit I:

(1) I.N. Herstein, Topics in Algebra.

(2) P.B. Bhattacharya, S.K. Jain, S. Nagpaul. Abstract Algebra.

Unit II: Cyclic groups and cyclic subgroups (15 Lectures)

(a) Cyclic subgroup of a group, cyclic groups, (examples including Z, Z_n and μ_n).

(b) Properties such as

- (i) Every cyclic group is abelian.
- (ii) Finite cyclic groups, infinite cyclic groups and their generators.
- (iii) A finite cyclic group has a unique subgroup for each divisor of the order of the group.
- (iv) Subgroup of a cyclic group is cyclic.
- (v) In a finite group G , $G =< a >$ if and only if $o(G) = o(a)$.
- (vi) If $G =< a >$ and $o(a) = n$ then $G =< a^m >$ if and only if $(m,\ n) = 1$.
- (vii) If G is a cyclic group of order p^n and $H < G,\ K < G$ then prove that either $H \subseteq K$ or $K \subseteq H$.

Reference for Unit II:

(1) I.N. Herstein, Topics in Algebra.

(2) P.B. Bhattacharya, S.K. Jain, S. Nagpaul. Abstract Algebra.

Unit III: Lagrange's Theorem and Group homomorphism (15 Lectures)

a) Definition of Coset and properties such as

1) If H is a subgroup of a group G and $x \in G$ then prove that
   - i) $xH = H$ if and only if $x \in H$.
   - ii) $Hx = H$ if and only if $x \in H$.
2) If H is a subgroup of a group G and $x,\ y \in G$ then prove that
   - i) $xH = yH$ if and only if $x^{-1}y \in H$.
   - ii) $Hx = Hy$ if and only if $xy^{-1} \in H$.
3) Lagrange's theorem and consequences such as Fermat's Little theorem, Eulers's theorem and If a group G has no nontrivial subgroups then order of G is a prime and G is Cyclic.

(b) Group homomorphisms and isomorphisms, automorphisms

(i) Definition.

(ii) Kernel and image of a group homomorphism.

(iii) Examples including inner automorphism.

Properties such as

1) $f : G \to G'$ is a group homomorphism then $Ker\, f < G$
   $f : G \to G'$ is a group homomorphism then
   $Ker\, f = \{e\}$ $if and\ only\ if\ f\ is$ $1-1$
   $f : G \to G'$ is an isomorphism of groups then
   - i) G is abelian if and only if G' is abelian.
   - ii) G is cyclic if and only if G' is cyclic.

Reference for Unit III:

(1) I.N. Herstein, Topics in Algebra.

(2) P.B. Bhattacharya, S.K. Jain, S. Nagpaul. Abstract Algebra.

Recommended Books:

1. I.N. Herstein. Topics in Algebra, Wiley Eastern Limited, Second edition.
2. N.S. Gopalakrishnan, University Algebra, Wiley Eastern Limited.
3. M. Artin, Algebra, Prentice Hall of India, New Delhi.
4. P.B. Bhattacharya, S.K. Jain, and S.R. Nagpaul, Abstract Algebra, Second edition, Foundation Books, New Delhi, 1995.
5. J.B. Fraleigh, A first course in Abstract Algebra, Third edition, Narosa, New Delhi.
6. J. Gallian. Contemporary Abstract Algebra. Narosa, New Delhi.

Additional Reference Books:

1. S. Adhikari. An Introduction to Commutative Algebra and Number theory. Narosa Publishing House.
2. T.W. Hungerford. Algebra, Springer.
3. D. Dummit, R. Foote. Abstract Algebra, John Wiley & Sons, Inc.
4. I.S. Luther, I.B.S. Passi. Algebra, Vol. I and II.

Suggested Tutorials:

1. Examples and properties of groups.
2. Group of symmetry of equilateral triangle, rectangle, square.
3. Subgroups.
4. Cyclic groups, cyclic subgroups, finding generators of every subgroup of a cyclic group.
5. Left and right cosets of a subgroup, Lagrange's Theorem.
6. Group homomorphisms, isomorphisms.

USMT 403/ UAMT 402: ORDINARY DIFFERENTIAL EQUATIONS

Unit I: First order First degree Differential equations (15 Lectures)

1. Definition of a differential equation, order, degree, ordinary differential equation and partial differential equation, linear and non linear ODE.
2. Existence and Uniqueness Theorem for the solutions of a second order initial value problem (statement only). Define Lipschitz function; solve examples verifying the conditions of existence and uniqueness theorem.
3. Review of solution of homogeneous and non- homogeneous differential equations of first order and first degree. Notion of partial derivative. Exact Equations: General Solution of Exact equations of first order and first degree. Necessary and sufficient condition for $Mdx + Ndy = 0$ to be exact. Non-exact equations. Rules for finding integrating factors (without proof) for non exact equations, such as i) $\frac{1}{Mx+Ny}$ is an I.F. if $Mx + Ny \neq 0$ and $Mdx + Ndy$ is homogeneous. ii) $\frac{1}{Mx-Ny}$ is an I.F. if $Mx - Ny \neq 0$ and $Mdx - Ndy$ is of the type $f_1(x,\ y)ydx + f_2(x,\ y)xdy$. iii) $e^{\int f(x)dx}\left(resp\ e^{\int g(y)dy}\right)$

is an I.F. if $N \neq 0$ $(resp\ M \neq 0)$ and $\frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right)$ $\left(resp\ \frac{1}{M}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right)\right)$ is a function of x $(resp\ y\ )$ alone, say $f(x)$ $\big(resp\ g(y)\big)$.

4. Linear and reducible to linear equations, finding solutions of first order differential equations of the type for applications to orthogonal trajectories, population growth, and finding the current at a given time.

Unit II: Second order Linear Differential equations (15 Lectures)

1. Homogeneous and non-homogeneous second order linear differentiable equations: The space of solutions of the homogeneous equation as a vector space. Wronskian and linear independence of the solutions. The general solution of homogeneous differential equation. The use of known solutions to find the general solution of homogeneous equations. The general solution of a non-homogeneous second order equation. Complementary functions and particular integrals.
2. The homogeneous equation which constant coefficient. auxiliary equation. The general solution corresponding to real and distinct roots, real and equal roots and complex roots of the auxiliary equation.
3. Non-homogeneous equations: The method of undetermined coefficients. The method of variation of parameters.

Unit III: Numerical Solution for Ordinary Differential Equations (15 Lectures)

Solution of Initial value problem of an ordinary first order differential equation:

1. One step methods; Taylor’s series method, Picard's method, Euler's method, Heun's method, Polygon method, Runge-Kutta method of second and fourth order; Accuracy of one-step methods.
2. Multistep methods (Predictor - Corrector methods).
3. Milne-Simpson method, Adams- Bashforth-Moulton method.
4. Accuracy of multistep methods.

Reference for Unit I and II:

(1) Differential equations with applications and historical notes, G. F. Simmons, McGraw Hill.

(2) An introduction to ordinary differential equations, E. A. Coddington Chapter 2: sections 7, 8,9,10 and Chapter 3: sections 14, 15, 16, 17,18,19,20 of G. F. Simmons. Also sections on similar topics from Coddington may be referred with problems only for second order and

simple third order equations.

Reference for Unit III: Chapter 13: sections 13.1, 13.2, 13.3, 13.4, 13.5, 13.6, 13.7, 13.8 and 13.9 of Numerical methods, E. Balaguruswamy, Tata-McGraw Hill.

Suggested Practicals (Tutorials for B. A.):

1. Application of existence and uniqueness theorem, solving exact and non exact equations.
2. Linear and reducible to linear equations, applications to orthogonal trajectories, population growth, and finding the current at a given time.
3. Finding general solution of homogeneous and non-homogeneous equations, use of known solutions to find the general solution of homogeneous equations.
4. Solving equations using method of undetermined coefficients and method of variation of parameters.
5. Taylor’s series method, Picard's method, Euler's method, Heun's method, Polygon method, Runge-kutta method of second order and calculating their accuracy.
6. Runge-Kutta method of fourth order, Milne-Simpson method, Adams- Bashforth-Moulton method and their accuracy.

Scheme of Examination for Semester III&IV

The performance of the learners shall be evaluated into two parts. The learner's performance shall be assessed by Internal Assessment with 25% marks in the first part by conducting the Semester End Examinations with 75% marks in the second part. The allocation of marks for the Internal Assessment and Semester End Examinations are as shown below:-

(a) Internal assessment 25 %

Courses with tutorials (Mathematics)

Sr No	Evaluation type	Marks
1	One class Test [Tutorial converted into test].s	20
2	Active participation in routine class instructional deliveries/Tutorials. Overall conduct as a responsible student, mannerism and articulation and exhibit of leadership qualities in organizing related academic actives.	05

(b) External Theory examination 75 %

1. Duration – The examinations shall be of 2.5 Hours duration.
2. Theory Question Paper Pattern:-There shall be four questions. Question number 1, 2 and 3 will be of 20 marks each (with internal options), while Question 4 will be of 15 marks (with internal options).
3. All questions shall be compulsory with internal choice within the questions.
4. Questions may be sub divided into sub questions as a, b, c, d & e, etc & the allocation of marks depends on the weightage of the topic.

Guidelines about conduct of Tutorials

Conduct and Evaluation: The tutorials should be conducted in batches formed as per the University circular. The tutorial session should consist of discussion between the teacher and the students in which students should participate actively. Each tutorial session and lectures should be evaluated out of 5 marks on basis of participation of student and the average of total aggregate should be taken.

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